

The Capacity of More Capable Cognitive Interference Channels

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Abstract—We explore fundamental limits of the discrete memoryless *cognitive interference channel* (DM-CIC) with two pairs of transmitter-receiver, in which the cognitive transmitter non-causally knows the message of the primary transmitter. It is known that superposition coding is optimal for several classes of cognitive channels, including the recently introduced *less noisy* DM-CIC [1]. In this work, we extend the results of *less noisy* DM-CIC to a broader class of DM-CIC known as *more capable* DM-CIC. Similar to the *less noisy* DM-CIC, two different *more capable* channels are conceivable: the *primary-more-capable* and *cognitive-more-capable* channels. We establish the capacity region for the latter case, i.e., when the cognitive receiver is more capable than the primary receiver, and we show that superposition coding is the optimal encoding technique. This new result is the largest capacity region for the DM-CIC to date, as all existing capacity results are explicitly shown to be its subsets.

I. INTRODUCTION

Wireless communication systems are limited in performance and capacity by interference. Despite having been studied for several decades, the capacity region of the simplest interference channel, i.e., two-user channel, is still an open problem. It is known only for a few special classes of interference channels, e.g., the strong and very strong interference [2].

With ever-increasing demand for radio spectrum, improving the spectral utilization in wireless communication systems is unavoidable. Cognitive radio is recognized as a key enabling technology for this purpose. Owing to the nodes which can sense the environment and adapt their strategy based on the network setup, cognitive radio technology is aimed at increasing the spectral efficiency in wireless communication systems. With this development in technology, networks with cognitive users are gaining prominence. Such a communication channel can be modeled by interference channel with cognition, simply known as the *cognitive channel* [3].

Most of the recent work on the cognitive interference channel has focused on the two user channel with one cognitive transmitter [3]–[13]. In this channel setting, one transmitter, known as the cognitive transmitter, has non-causal access to the message transmitted by the other transmitter (the primary transmitter). This can be used to model an “ideal” cognitive radio. We study the two-user *discrete memoryless* cognitive interference channel (DM-CIC), too.

Fundamental limits of the DM-CIC, in which the cognitive transmitter non-causally knows the the full message of the the

primary user, have been explored for several years now. The capacity of this channel remains unknown except for some special classes, e.g., in the “weak interference” [5], “strong interference” [7], and “cognitive less noisy” [1] regimes.

In this paper, we extend the work on *less noisy* DM-CIC [1] to the *more capable* DM-CIC. The notion of more capable DM-CIC first appeared in [10], in which the primary receiver is more capable than the secondary one. We observe that, similar to the *less noisy* DM-CIC, two different *more capable* cognitive channels are conceivable: the *primary-more-capable* and *cognitive-more-capable* DM-CIC. The former was studied in [10], [11]; the latter is the subject of study in this work.

The main contribution of this paper is to establish a new capacity result for the DM-CIC, i.e., capacity region for the cognitive-more-capable DM-CIC. To this end, we first propose a new outer bound for this channel; the outer bound is developed from the outer bound introduced in [5, Theorem 3.2]. We then show that this outer bound is the same as an inner bound which is based on superposition coding. Therefore, we characterize the capacity region for the cognitive-more-capable DM-CIC and prove that superposition coding is the capacity-achieving technique.

In the second part of this paper, we prove that this new capacity result is the largest capacity region for the DM-CIC to date. We explicitly show that the existing capacity results are subsets of this new capacity region. In fact, the capacity of the cognitive-more-capable DM-CIC reduces to the capacity regions in the “weak interference” [5], “strong interference” [7], and “less noisy” [1] regimes once the corresponding channel conditions are satisfied. Finally, the relation among different capacity results of the DM-CIC is clarified in light of this work and [14].

The rest of the paper is organized as follows. The system model and definitions are presented in Section II. Section III provides the main result of this paper, which includes the capacity region for the cognitive-more-capable DM-CIC. In Section IV, we show that the new capacity result includes all existing capacity results as subsets. This is followed by conclusions in Section V.

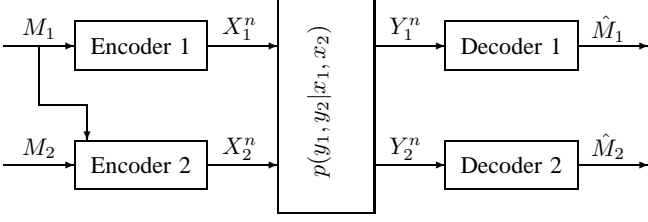


Fig. 1. The discrete memoryless cognitive interference channel (DM-CIC) with two transmitters and two receivers. M_1, M_2 are two messages, X_1, X_2 are inputs, Y_1, Y_2 are outputs, and $p(y_1, y_2 | x_1, x_2)$ is the transition probability of channel.

II. PRELIMINARIES AND DEFINITIONS

The two-user discrete memoryless cognitive interference channel (DM-CIC) is an interference channel [15] that consists of two transmitter-receiver pairs, in which one transmitter (the cognitive user) knows the message of the other transmitter (the primary one), in addition to its own message. In what follows, we formally define this channel and a special class of that.

A. Discrete memoryless cognitive interference channel

The discrete memoryless cognitive interference channel (DM-CIC) is depicted in Figure 1. Let M_1 and M_2 be two independent messages which are uniformly distributed on the set of all messages for the first and second users, respectively. Transmitter i wishes to transmit message M_i to receiver i , in n channel use at rate R_i , and $i = 1, 2$. Message M_2 is available only at transmitter 2, while both transmitters know M_1 . This channel is defined by a tuple $(\mathcal{X}_1, \mathcal{X}_2; p(y_1, y_2 | x_1, x_2); \mathcal{Y}_1, \mathcal{Y}_2)$ where $\mathcal{X}_1, \mathcal{X}_2$ and $\mathcal{Y}_1, \mathcal{Y}_2$ are input and output alphabets, and $p(y_1, y_2 | x_1, x_2)$ is channel transition probability density functions.

The capacity of the DM-CIC is known in the “cognitive less noisy” [1], “strong interference” [4], “weak interference” [5], and “better cognitive decoding” [9] regimes. These capacity results are listed in Table I, and labeled $\mathcal{C}_I, \mathcal{C}_{II}, \mathcal{C}_{III}$, and \mathcal{C}'_{III} , respectively. In all above cases, the cognitive receiver has a better condition (more information) than the primary one in some sense, as it can be understood from the corresponding conditions in Table I. In the second case, both receivers can decode both messages.

B. More Capable DM-CIC

Since the second transmitter has complete and non-causal knowledge of both messages, it can act like a BC transmitter. Particularly, in the absence of the first transmitter this channel becomes the well-known DM-BC [16]. In the presence of that, this channel is no longer a BC; however, one can define conditions, similar to that in the BC, showing that one receiver is in a “better” condition than the other to decode the

messages, e.g., one receiver is *less noisy* or *more capable* than the other [17].

In [10], [11], the authors extended this notion to the DM-CIC, and studied the case where the primary receiver is more capable than the cognitive receiver. This led to the capacity of GCZIC at very strong interference. In what follows, we show that similar to the less noisy DM-CIC, and depending on which receiver is in a better condition than the other, two different more capable DM-CIC arises. These two are formally defined in the following.

Definition 1. The DM-CIC is said to be *primary-more-capable*¹ if

$$I(X_1, X_2; Y_1) \geq I(X_1, X_2; Y_2) \quad (1)$$

for all $p(x_1, x_2)$.

Definition 2. The DM-CIC is said to be *cognitive-more-capable* if

$$I(X_1, X_2; Y_2) \geq I(X_1, X_2; Y_1) \quad (2)$$

for all $p(x_1, x_2)$.

It can be noted that in the first case the primary receiver has more information about transmitted codewords, than the cognitive receiver whereas the reverse is true in the second case. Therefore, given the channel condition, a DM-CIC can be either in the *primary-more-capable* or in the *cognitive-more-capable* regimes. The former was studied in [10], [11]. In this paper, we focus on the latter case.

III. MAIN RESULTS

In this section, we first introduce a new outer bound on the capacity of the cognitive-more-capable DM-CIC. We then find an alternative representation of this outer bound; the new representation is the same as an achievable rate region for the DM-CIC which is based on superposition coding. Consequently, the capacity region of the cognitive-more-capable DM-CIC is established.

A. New Outer Bounds

The following provides an outer bound on the capacity of the cognitive-more-capable DM-CIC, defined in (2).

Theorem 1. Define \mathcal{R}_o as the set of all rate pairs (R_1, R_2) such that

$$R_1 \leq I(U, X_1; Y_1), \quad (3a)$$

$$R_1 + R_2 \leq I(U, X_1; Y_1) + I(X_2; Y_2 | U, X_1), \quad (3b)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_2), \quad (3c)$$

for the probability distribution $p(u, x_1, x_2)p(y_1, y_2 | x_1, x_2)$. Then, for some $p(u, x_1, x_2)$, \mathcal{R}_o provides an outer bound on the capacity region of the cognitive-more-capable DM-CIC, defined by (2).

¹This definition first appeared in [10], [11] under the name of the “more capable” DM-CIC.

Proof: The proof is provided in Section VI-A. ■

Theorem 2. Let \mathcal{R}'_o be the set of all rate pairs (R_1, R_2) such that

$$R_1 \leq I(U, X_1; Y_1), \quad (4a)$$

$$R_2 \leq I(X_2; Y_2|U, X_1), \quad (4b)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_2), \quad (4c)$$

for the probability distribution $p(u, x_1, x_2)p(y_1, y_2|x_1, x_2)$. Then $\mathcal{R}'_o \equiv \mathcal{R}_o$, that is, \mathcal{R}'_o gives an outer bound on the capacity region of the cognitive-more-capable DM-CIC, for some $p(u, x_1, x_2)$.

Proof: To prove this we consider the following two cases: cases 1: when (3b) is redundant in \mathcal{R}_o , i.e.,

$$I(U, X_1; Y_1) + I(X_2; Y_2|U, X_1) \geq I(X_1, X_2; Y_2). \quad (5)$$

cases 2: when (3c) is redundant in \mathcal{R}_o , i.e.,

$$I(U, X_1; Y_1) + I(X_2; Y_2|U, X_1) \leq I(X_1, X_2; Y_2). \quad (6)$$

In the first case, we can see that (4b) becomes redundant also. This is because the right-hand side of (4b) is greater than or equal to the difference between the right-hand sides of (4c) and (4a), owing to (5). Consequently, the remaining constraints in \mathcal{R}_o and \mathcal{R}'_o are the same, and $\mathcal{R}'_o \equiv \mathcal{R}_o$.

In the second case, it is obvious that the third inequality is redundant both in (3) and (4). Therefore, the set of constraints in Theorem 1 reduces to

$$R_2 \leq I(U, X_2; Y_2), \quad (7a)$$

$$R_1 + R_2 \leq I(U, X_2; Y_2) + I(X_1; Y_1|U, X_2). \quad (7b)$$

Similarly, the set of constraints in Theorem 2 reduces to

$$R_1 \leq I(X_1; Y_1|U, X_2), \quad (8a)$$

$$R_2 \leq I(U, X_2; Y_2). \quad (8b)$$

Let \mathcal{R}_{o1} denote the union of all rate pairs (R_1, R_2) that satisfy (7a)-(7b) and \mathcal{R}'_{o1} be the union of all rate pairs (R_1, R_2) that satisfy (8a)-(8b); we show that $\mathcal{R}_{o1} \equiv \mathcal{R}'_{o1}$. To this end, using the same argument as El Gamal [18], we can see that any point on the boundary of \mathcal{R}'_{o1} is also on the boundary of \mathcal{R}_{o1} . In fact, as stated in [17], \mathcal{R}'_{o1} is an alternative characterization of \mathcal{R}_{o1} for any $p(u, x_1, x_2)$. This is because R_2 can be thought of as the rate of common message that can be decoded at both receivers while R_1 is the rate of the private message. Now $(R_2, R_1) \in \mathcal{R}'_{o1}$ if and only if $(R_2 - t, R_1 + t) \in \mathcal{R}'_{o1}$ for any $0 \leq t \leq R_2$. In other words, the common rate R_2 can be partly or wholly private. Thus region \mathcal{R}'_{o1} can be represented as \mathcal{R}_{o1} , i.e., $\mathcal{R}_{o1} \equiv \mathcal{R}'_{o1}$.

Therefore, the proof of Theorem 2 is completed as in the both cases $\mathcal{R}'_o \equiv \mathcal{R}_o$. ■

It should be indicated that the outer bounds in Theorem 1 and Theorem 2 are valid only for the cognitive-more-capable DM-CIC, defined in (2). However, if we remove the third inequalities ((3c) and (4c)) in those sets of inequalities, the remaining constraints in each set provide outer bounds for any DM-CIC.

B. An Achievable Rate Region

The following theorem gives an achievable rate regions for the DM-CIC.

Theorem 3. The union of rate regions given by

$$R_1 \leq I(W, X_1; Y_1), \quad (9a)$$

$$R_2 \leq I(X_2; Y_2|W, X_1), \quad (9b)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_2), \quad (9c)$$

is achievable for the DM-CIC, where the union is over all probability distributions $p(w, x_1, x_2)$.

Proof: This achievable region uses superposition encoding at the cognitive transmitter, as the cognitive user knows the primary user's message). The proof of Theorem 3 uses the superposition coding idea in which Y_1 can only decode M_1 while Y_2 (the more capable receiver) is intended to decode both M_1 and M_2 . Considering the space of all codewords, one can view the (W, X_1) as *cloud centers*, and the X_2 as *satellites* [19]. The decoding is based on joint typicality. The details of the proof can be found in [1]. ■

Remark 1. The achievable region in Theorem 3 is a subset of the achievable region in [9, Theorem 7]. This can be shown by setting $U = U_{1c}, X_1 = U_{1pb}, X_2 = U_{2c} = U_{2pb}$ and using the Fourier-Motzkin elimination to simplify the region. Note that the indices 1 and 2 need to be swapped. Further details can be found in Section VI-B.

C. The Capacity of the Cognitive-more-capable DM-CIC

The capacity region of the cognitive-more-capable DM-CIC is established immediately in light of the outer bound in Theorem 2 and the inner bound in Theorem 3. That is, the region define by \mathcal{R}'_o , or equivalently the rate region characterized in Theorem 3, gives the capacity region of the DM-CIC when (2) holds.

Theorem 4. For the cognitive-more-capable DM-CIC defined in (2), the capacity region is given by the set of all rate pairs (R_1, R_2) such that

$$R_1 \leq I(W, X_1; Y_1), \quad (10a)$$

$$R_2 \leq I(X_2; Y_2|W, X_1), \quad (10b)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_2), \quad (10c)$$

for some $p(w, x_1, x_2)$.

Theorem 4 gives the largest capacity region for the DM-CIC channel to date. We next prove that this capacity result contains all existing capacity results for the DM-CIC channel.

IV. COMPARISON AND CLASSIFICATION

In this section, we compare the capacity region obtained in Theorem 4 with all of the previously known capacity results for the DM-CIC. For ease of comparison, these results are summarized in Table I. We show that the capacity region of the cognitive-more-capable DM-CIC contains all other capacity regions listed in Table I, as subsets. We also clarify the relation

TABLE I

SUMMARY OF EXISTING AND NEW CAPACITY RESULTS FOR THE DISCRETE MEMORYLESS COGNITIVE INTERFERENCE CHANNEL (DM-CIC). THE SUBSCRIPTS 1 AND 2, RESPECTIVELY, DENOTE THE PRIMARY AND SECONDARY (COGNITIVE) USERS.*

Label	DM-CIC class	Condition	Capacity region	Reference
\mathcal{C}_I	cognitive-less-noisy	$I(U; Y_1) \leq I(U; Y_2)$	$R_1 \leq I(U; Y_1)$ $R_2 \leq I(X_2; Y_2 U)$	[1]
\mathcal{C}_{II}	strong interference	$I(X_1, X_2; Y_1) \leq I(X_1, X_2; Y_2)$ $I(X_2; Y_2 X_1) \leq I(X_2; Y_1 X_1)$	$R_1 \leq I(X_1; Y_1)$ $R_2 \leq I(X_2; Y_2 X_1)$	[7]
\mathcal{C}_{III}	weak interference	$I(X_1; Y_1) \leq I(X_1; Y_2)$ $I(U; Y_1 X_1) \leq I(U; Y_2 X_1)$	$R_1 \leq I(U, X_1; Y_1)$ $R_2 \leq I(X_2; Y_2 U, X_1)$	[5]
\mathcal{C}'_{III}	better-cognitive-decoding	$I(U, X_1; Y_1) \leq I(U, X_1; Y_2)$	$R_1 \leq I(U, X_1; Y_1)$ $R_2 \leq I(X_2; Y_2 X_1)$ $R_1 + R_2 \leq I(U, X_1; Y_1) + I(X_2; Y_2 U, X_1)$	[9]
\mathcal{C}_{IV}	cognitive-more-capable	$I(X_1, X_2; Y_1) \leq I(X_1, X_2; Y_2)$	$R_1 \leq I(U, X_1; Y_1)$ $R_2 \leq I(X_2; Y_2 U, X_1)$ $R_1 + R_2 \leq I(X_1, X_2; Y_2)$	Theorem 4

* It should be emphasized that $\mathcal{C}'_{III} \equiv \mathcal{C}_{III}$ [14], $\mathcal{C}_I \subseteq \mathcal{C}_{III} \subseteq \mathcal{C}_{IV}$ and $\mathcal{C}_{II} \subseteq \mathcal{C}_{IV}$.

between the other capacity results. More precisely, we prove that $\mathcal{C}_I \subseteq \mathcal{C}_{III} \equiv \mathcal{C}'_{III} \subseteq \mathcal{C}_{IV}$ and $\mathcal{C}_{II} \subseteq \mathcal{C}_{IV}$. The conditions required for \mathcal{C}_I and \mathcal{C}_{II} do not imply each other, in general. However, we should emphasize that a DM-CIC can be in all of those regimes, simultaneously. This happens, for example, when $Y_1 = Y_2$.

We first show that the conditions required for the strong interference and cognitive-less-noisy regimes cannot imply each other. This is because, in the cognitive-less-noisy DM-CIC, $I(U; Y_1) \leq I(U; Y_2)$ holds for any $p(u, x_1, x_2)$ which results in $I(U; Y_1|X_1) \leq I(U; Y_2|X_1)$ for all $p(u, x_1, x_2)$. Then for $U = X_2$ we obtain $I(X_2; Y_1|X_1) \leq I(X_2; Y_2|X_1)$. Since in the strong interference the channel must satisfy the last inequality with the “reverse” sign, we conclude that a DM-CIC cannot be in the cognitive-less-noisy and strong interference regimes at the same time, except for the trivial case $Y_1 = Y_2$. Hence, excluding the trivial case $Y_1 = Y_2$, these two regimes are mutually exclusive. It can be also shown that for a DM-CIC the followings hold:

- 1) The better-cognitive-decoding and weak interference are equivalent (see [14]).²
- 2) If a DM-CIC is in the strong interference regime then it is in the cognitive-more-capable regime, as well.
- 3) If a DM-CIC is in the cognitive-less-noisy regime then it is in the better-cognitive-decoding regime.
- 4) A better-cognitive-decoding DM-CIC is cognitive-more-capable, too.

²In [14] it is shown that the conditions required for a DM-CIC to be in the “better-cognitive-decoding” and “weak interference” regimes, as well as their corresponding capacity regions are equal. As a result, we can simplify representation of these two capacity results in the following form.

Lemma 1. For a DM-CIC satisfying $I(U, X_1; Y_1) \leq I(U, X_1; Y_2)$, the capacity region is given by the set of all rate pairs (R_1, R_2) such that

$$R_1 \leq I(U, X_1; Y_1), \quad (11a)$$

$$R_2 \leq I(X_2; Y_2|U, X_1), \quad (11b)$$

for some $p(u, x_1, x_2)$.

For completeness we show this in Section VI-C.

DM-CIC

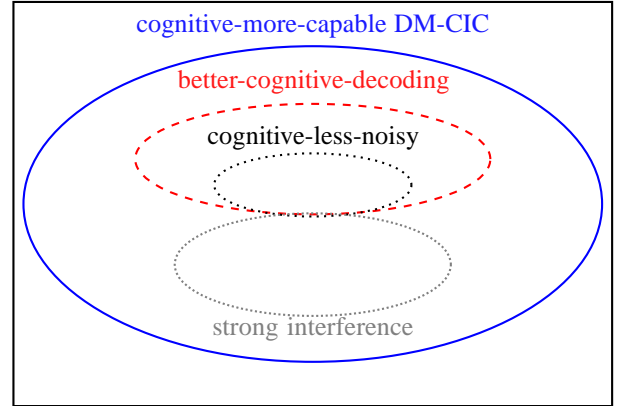


Fig. 2. The class of the discrete memoryless cognitive interference channels (DM-CIC). The cognitive receiver is superior than the primary receiver for the cognitive-more-capable and all its subclasses. The largest ellipse (blue, solid line) represents the cognitive-more-capable DM-CIC. The ellipse with red, dashed lines represents the better-cognitive-decoding DM-CIC; note that, this regime is equivalent to weak interference regime. The other two ellipses, i.e., dotted and densely dotted ellipses, respectively show the cognitive-less-noisy and strong interference DM-CIC. The area between the largest ellipse and rectangle corresponds to the case that the primary receiver is more capable than cognitive receiver; this is studied in [10], [11].

The last two statements are evident by their corresponding conditions in Table I because

$$\begin{aligned}
 &I(U; Y_1) \leq I(U; Y_2) \quad \forall p(u) \\
 &\Rightarrow I(U, X_1; Y_1) \leq I(U, X_1; Y_2) \quad \forall p(u, x_1) \\
 &\Rightarrow I(X_2, X_1; Y_1) \leq I(X_2, X_1; Y_2) \quad \forall p(u, x_1, x_2).
 \end{aligned}$$

The second statement is also obvious as strong interference impose one extra condition compared to the cognitive-more-capable regime. Note that, the converse of statements 2, 3, and 4 does not hold in general. Figure 2 represents these relations, pictorially.

In light of the above classifications, the constraints characterizing the capacity region of the cognitive-more-capable

DM-CIC (i.e., \mathcal{C}_{IV}) can be used to represent the capacity region of all other classes of the DM-CIC listed in Table I. Furthermore, when the condition corresponding to each one of those subclasses holds, \mathcal{C}_{IV} reduces to the corresponding capacity result.³ For one thing, if a cognitive-more-capable DM-CIC further satisfies $I(U, X_1; Y_1) \leq I(U, X_1; Y_2)$ then it is easy to see that the third constraint in \mathcal{C}_{IV} is redundant, and the capacity region corresponding to “better-cognitive-decoding,” or equivalently “weak interference,” is achieved (see Lemma 1). The other two regions (\mathcal{C}_I , \mathcal{C}_{II}) are also attainable from \mathcal{C}_{IV} ; details can be found in [1] and [14].

In summary, from the above arguments we conclude that

- \mathcal{C}_I and \mathcal{C}_{II} are disjoint (except when $Y_1 = Y_2$)
- $\mathcal{C}_I \subseteq \mathcal{C}_{III} \equiv \mathcal{C}'_{III} \subseteq \mathcal{C}_{IV}$
- $\mathcal{C}_{II} \subseteq \mathcal{C}_{IV}$

Remark 2. Superposition coding is optimal for several classes of the DM-CIC for which the cognitive receiver is superior than the primary one, as we detailed in this section. It is, however, not optimal in general since requiring the cognitive receiver to recover both messages (even though non-uniquely for the primary’s message) can excessively constraint the achievable rate region.

Remark 3. All of the classes defined in Table I and depicted in Figure 2, imply the superiority of the cognitive receiver the primary one. It is worth noting that, by swapping the indices 1 and 2 in the conditions, we can define similar classes in which the primary receiver is superior than the cognitive one. One may expect similar capacity results in the new cases by using superposition encoding in a different order. But it cannot come true because the factorization of probability distribution $p(u, x_1, x_2)$ is different since the primary encoder does not know x_2 ; thus, similar rate regions are not attainable over general distribution $p(u, x_1, x_2)$. This has been noted in [10].

V. CONCLUSIONS

We have established the capacity of a new class of DM-CIC, named the cognitive-more-capable DM-CIC, which gives the largest capacity region for the DM-CIC up to now. This is proved by showing that all previously known capacity regions, for the DM-CIC, are subsets of this new result, which is achieved by using superposition coding at the cognitive transmitter. The analysis of the other capacity results of the DM-CIC shows that superposition coding is the capacity-achieving techniques in those cases, too. Finally, we make a logical link between the different capacity results for this channel. This sheds more light on the existing capacity results and unifies all of them under the capacity region in the new regime.

³To better appreciate this, we may think of the two well-known “less noisy” and “more capable” BC and the relation between their capacity. We know that the condition required for a less noisy BC implies that of more capable BC [17]. That is, any less noisy BC is more capable also. Hence, the capacity of more capable BC reduces to that of less noisy BC once the condition required for less noisy BC is met. This is clear from the capacity regions of these two channels [17].

VI. APPENDIX

A. Proof of Theorem 2

Proof: The first two inequalities in this outer bound (i.e., (3a), (3b)), which make an outer bound on the capacity of any DM-CIC, are proved in [5, Theorem 3.2]. Here, we show that the last constraint (3c) holds for a DM-CIC that satisfies (2). The proof follows the same footstep as the proof of the last inequality in [11, Theorem 2]; the only difference is that [11, Theorem 2] is proved for the primary-more-capable DM-CIC while we are dealing with the cognitive-more-capable DM-CIC. Hence, it suffices to swap the indices 1 and 2 in that proof. This can also be found in [4]. ■

B. Proof of Remark 1

Proof: As Rini et al. mention in the proof of [9, Theorem 10], for

$$U = U_{1c}, \quad (12a)$$

$$X_1 = U_{1pb}, \quad (12b)$$

$$X_2 = U_{2c} = U_{2pb}, \quad (12c)$$

their achievable rate region in [9, Theorem 7] reduces to

$$R_1 \leq I(U, X_1; Y_1 | X_2), \quad (13a)$$

$$R_2 \leq I(U, X_2; Y_2), \quad (13b)$$

$$R_1 + R_2 \leq I(U, X_2; Y_2) + I(X_1; Y_1 | U, X_2), \quad (13c)$$

$$R_1 + R_2 \leq I(U, X_2, X_1; Y_1), \quad (13d)$$

after Fourier-Motzkin elimination [9]⁴.

We show that (13a) is redundant. To this end, we first see that

$$I(X_1; Y_1 | U, X_2) \leq I(U, X_1; Y_1 | X_2), \quad (14)$$

which is obvious by applying chain rule to the right-hand side and nonnegativity of information. Now, we use (14) to show that the first constraint in the set of inequalities given by (13) is redundant. This is proved by showing that the rate constraint (13a) is always loose since the right-hand side of (13a) is greater than or equal to the difference between the right-hand sides of (13c) and (13b), which is obvious owing to (14). Therefore, the set of constraints in (13) reduces to

$$R_1 \leq I(U, X_2; Y_2), \quad (15a)$$

$$R_1 + R_2 \leq I(U, X_2; Y_2) + I(X_1; Y_1 | U, X_2), \quad (15b)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_1), \quad (15c)$$

where we have also simplified the right-hand side of (13d) as $I(U, X_2, X_1; Y_1) = I(X_2, X_1; Y_1)$ follows by the Markov chain $U \rightarrow X_2, X_1 \rightarrow Y_1$.

Finally, by swapping indices 1 and 2, we can see that the achievable rate region in (15) is the same as outer bound in Theorem 1. On the other hand, in Section III we proved that the set of inequalities in Theorem 2 gives an alternative representation for that in Theorem 1. Hence (15) is also another representation for the achievable rate region in Theorem 3. ■

⁴Note that, compared to our setting, the indices are swapped in [9].

C. Proof of Lemma 1

Proof: We first define the conditions required for these two regimes. The DM-CIC is said to be in the “weak interference” regime [9, eq. (6)] if

$$I(U; Y_1|X_1) \leq I(U; Y_2|X_1), \quad (16a)$$

$$I(X_1; Y_1) \leq I(X_1; Y_2), \quad (16b)$$

for all $p(u, x_1, x_2)$.

Likewise, the DM-CIC is said to be in the “better cognitive decoding” regime [9, eq. (15)] if

$$I(U, X_1; Y_1) \leq I(U, X_1; Y_2), \quad (17)$$

for all $p(u, x_1, x_2)$.

Now, we prove that the “better cognitive decoding” condition in (16) is equivalent to the “weak interference” condition in (17). We show that that (16) implies (17) and vice versa. To prove the direct implication, we show that (17) implies both inequalities (16a) and (16b). The latter inequality is achieved by setting $U = \emptyset$ in (17). To prove the former, similar to [7, Lemma 5], we can write

$$\begin{aligned} I(U; Y_1|X_1) &= I(U, X_1; Y_1|X_1) \\ &= \sum_{x_1} p(x_1) I(U, X_1; Y_1|X_1 = x_1) \\ &\leq \sum_{x_1} p(x_1) I(U, X_1; Y_2|X_1 = x_1) \quad (18) \\ &= I(U, X_1; Y_2|X_1) \\ &= I(U; Y_2|X_1), \end{aligned}$$

in which the inequality follows because $I(U, X_1; Y_1) \leq I(U, X_1; Y_2)$ holds for all joint distributions $p(u, x_1, x_2)$. This completes the proof of the direct part, i.e., (16) \Rightarrow (17).

The proof of the converse part is rather simple; it suffices to add up the inequalities (16a) and (16b) to get (17). Therefore, both directions are established; i.e., (17) \Leftrightarrow (16). This means that the “better cognitive decoding” condition in (17) and the “weak interference” condition in (16) are essentially the same. Consequently, their corresponding capacity results must be equal as well. This completes the proof for Lemma 1 by using the condition from (17) and capacity region representation corresponding to (16). ■

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